Mechanics of Particle Bounce

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Irregular particles bounce with randomly distributed angles of reflection. There is a certain probability that a particle striking at low angle will bounce at a high angle and be carried far out into the main fluid flow. Bounce phenomena were investigated with respect to bounce of a model particle, limitations on dust collection (back mixing), and energy loss during the flow of suspensions.

Particles with irregular shapes and mass distributions no not necessarily reflect when they strike a surface but bounce at some angle with a probability distribution about the average angle of reflection. During bounce, energy interchange can occur between particle and surface (inelastic bounce) and between the translational and rotational energies of the particle. The process is complicated further when the surface struck is itself irregular in shape and when friction occurs between particle and surface at the instant of impact.

There are practical consequences of irregular bounce. In dust collectors a particle hitting at a high angle of incidence has a certain probability of bouncing at a lower angle of reflection and at a higher lateral velocity. In this case irregular bounce limits efficiency by causing back mixing. During pneumatic conveying, irregular bounce keeps large particles suspended and moving. Here the lateral velocity of a high bounce represents kinetic energy lost because the particle must be reaccelerated axially, and the high bounce itself contributes to a general lagging of the particle behind the gas flow.

It is the purpose of this paper to discuss simply the mechanics of particle bounce and to show the importance of bounce phenomena in certain applications.

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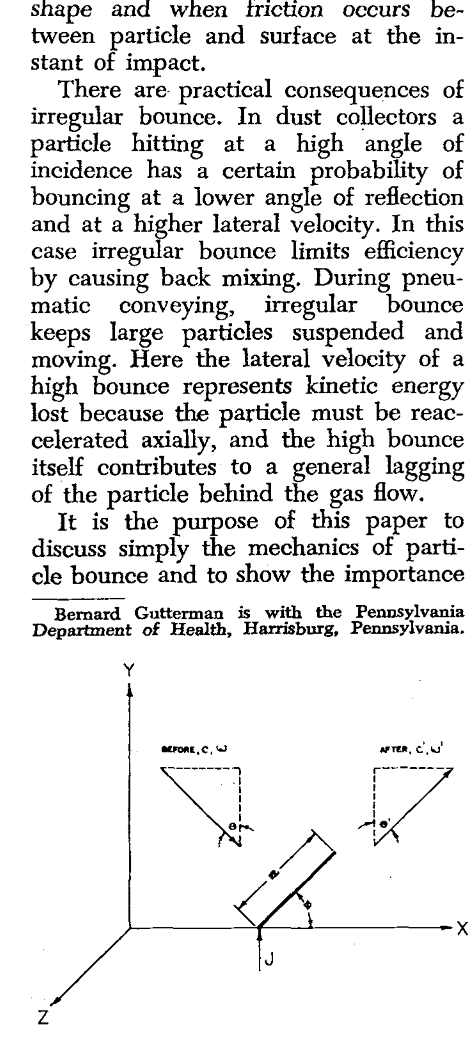

Fig. 1. Graphical representation of bouncing stick: θ = angle of incidence, θ' = angle of reflection, ϕ = angle stick makes with surface at moment of impact.

BOUNCING STICK

To establish the nature of the theory of irregular bounce and show in a simple mathematical way, the interchange of kinetic and rotational energies, and the probability of various angles of bounce, a simple theoretical model was devised. This model involves the initial bounce of a uniformly weighted line segment or stick. It is assumed that collision is between smooth, rigid surfaces and that the motion is two-dimensional with rotation in the third dimension. Figure 1 is a graphical representation of this bouncing stick. The angle made by the incident path of the center of mass with a normal to the surface is defined as the *angle of incidence*; the angle made by the reflected path of the center of mass with a normal to the surface is defined as the *angle of reflection*.

At the moment of impact the linear momentum imparted to the stick is

$$m(\mathbf{C}' - \mathbf{C}) = (\mathbf{x} + \mathbf{y}) \cdot \mathbf{J} \qquad (1)$$

When there is no friction, that is, the impulsive change of momentum is upward, the linear momentum imparted in the y direction is

$$m(C'_y - C_y) = J \qquad (2)$$

and in the x direction

$$m(C'_x - C_x) = 0 \qquad (3)$$

an indication that there is no change in the velocity tangential to the surface.

For rotation about the z axis the angular momentum is $(\mathbf{I}\cdot\omega)_z = I_{zz}\omega_z$, and at the moment of impact the angular momentum imparted to the stick is

$$I(\omega' - \omega) = J\,\mathcal{L}\cos\phi \qquad (4)$$

Since an "elastic" collision between smooth, rigid surfaces is assumed, energy is conserved, and

$$E = (1/2)m\mathbf{C}^2 + (1/2)\omega\cdot\mathbf{I}\cdot\omega$$
$$= (1/2)m(C^2_x + C^2_y) + (1/2)I\omega^2$$
$$= E' = (1/2)m(C'^2_x + C'^2_y) + (1/2)I\omega'^2 \qquad (5)$$

Solving for J from Equations (2) through (5), one gets

$$J = -2mC_y(1 + (\omega\mathcal{L}/C_y)\cos\phi)/(1 + m\mathcal{L}^2\cos^2\phi/I) \qquad (6)$$

Equations (2) and (6) give C'_y in terms of C_y and ω. Equations (4) and (6) give ω' in terms of C_y and ω.

Of particular interest are the relationships between θ and θ', between initial and final vertical velocities, and between initial and final kinetic energies. A general relationship indicating all these changes can be written as

$$\frac{\cot\theta'}{\cot\theta} = \frac{C'_y}{C_y} = 1 - \frac{2(1 + (\omega\mathcal{L}/C_y)\cos\phi)}{1 + 3\cos^2\phi} \qquad (7)$$

Figure 2 shows a plot of these ratios as a function of stick orientation at the moment of impact. Equation (7) expresses only the result of a first collision and shows the change in the trajectory angle and velocity of the center of mass. Curves in Figure 2 are drawn for $\omega\mathcal{L}/C_y = 0$, a nonrotating stick, and

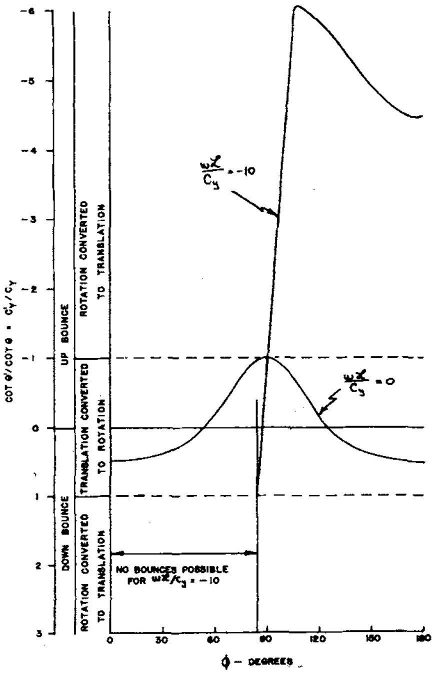

Fig. 2. Result of single collision of bouncing stick.

$\omega\mathcal{L}/C_y = -10$; the latter case is one where the velocity tangential to the surface is perhaps ten times that of the vertical velocity toward the surface, and a peripheral velocity equal to this velocity has been imparted by previous bounces.

Positive values of $\cot\theta'/\cot\theta$ indicate that the center of mass of the stick, after the initial impact, continues to move downward toward the surface. Such motion would always lead to a second impact before rebound. When $\cot\theta'/\cot\theta$ is negative, the center of mass moves upward after impact. When $\cot\theta'/\cot\theta$ is less than -1, the stick "hops" and rebounds at a higher angle and at a higher vertical velocity than the angle and velocity with which it hit. If the stick hits end down ($\phi =$ 90 deg.), $\cot\theta'/\cot\theta = -1$, and the stick reflects perfectly without any change in kinetic or rotational energy. When the absolute value of $\cot\theta'/\cot\theta$ $= C'_y/C_y$ is less than unity, and since $C'_x = C_x$, kinetic energy is interchanged for rotational energy at the moment of impact. When the absolute value is greater than unity, the energy interchange is reversed.

If the absolute value of $\omega\mathcal{L}/C_y$ is greater than one, the rotation is great enough to cause the tip of the stick to move vertically faster than the center of the mass. This will decrease the possible orientation angles (ϕ) from 180 deg. to some smaller number. Mathematically this restricts J to be negative as calculated by Equation (6). Thus the following must be true:

$$\frac{\omega\mathcal{L}}{C_y}\cos\phi_o \geqq -1$$

For the example mentioned $\omega\mathcal{L}/C_y =$ -10; $\phi_c = 84$ deg. 16 min. This is to say that no bounces occur with $\phi < 84$ deg. 16 min. when $\omega\mathcal{L}/C_y = -10$. Here ω is clockwise, negative; C_y is downward, positive. From Figure 2 it can be seen that 97% of the sticks will bounce upward after the first impact. It is also apparent that a significant number of these upward motions will be at lower angles of reflection than the angle of incidence and at larger upward velocities than the initial downward velocity, all brought about by the irregularity in shape.

To show this more strikingly, average values are computed. If all possible orientation angles ϕ are equally probable, the average $\cot\theta'$ and the average C'_y for $\omega\mathcal{L}/C_y = -10$ is given by

$$\frac{\overline{\cot\theta'}}{\cos\theta} = \frac{\overline{C'_y}}{C_y} = \frac{180\text{ deg.}}{\pi[180 - 84\text{ deg. }16\text{ min.}]}\int_{84\text{ deg. }16\text{ min.}}^{180\text{ deg.}} \left\{1 - \frac{2(1-10\cos\theta)}{1+3\cos^2\theta}\right\} d\theta = -4.50 \quad (8)$$

BOUNCE OF AN IRREGULAR MODEL PARTICLE

To demonstrate the bounce characteristics of irregularly shaped particles, a simple model test was devised. Ping-pong balls, distorted by boiling, were shot from a gun against a hard surface at a particular incident angle, and the distribution of ultimate angle of reflection was measured.

Incident angles were varied from 80 to 50 deg., and reflected angles were obtained by markings through carbon paper at points of contact on the bounce surface, which was horizontal, and a vertical backboard. Five representative shapes were fired about fifty times each at a constant incident angle. These shapes showed a high degree of correlation with each other, although by visual standards they were entirely different in geometry.

When the data from different incident angles were assembled, it was found that the standard deviation from the angle of reflection appeared to vary with the incident angle. As shown in Figure 3, the standard deviation from the average angle of reflection varied from approximately 9 to 10 deg. for an incident angle of 50 deg. to 5 to 6 deg. for an incident angle of 80 deg. For incident angles greater than 70 deg., the mean reflected angle was approximately equal to the incident angle, and the distribution of angular deviations was normal in shape. A sample distribution is shown in Figure 4.

Results from the bounce of a soft rubber cube on a smooth varnished wood surface and a hard Lucite cube against a smooth steel surface are also shown in Figure 3. Apparently friction plays an important role in setting the standard deviation. With rubber cubes the average angle of reflection was found to be lower than the angle of incidence for high angles of incidence and higher for low angles, the difference being more pronounced at high angles of incidence. With Lucite cubes the average angle of reflection was always more than the angle of incidence, the difference slightly increasing with decreasing angle of incidence.

BACK MIXING CAUSED BY PARTICLE BOUNCE

Large particles conveyed in horizontal gas flows remain suspended even though they are not much affected by turbulent gas motion and should, because of the gravity force, be rolling along the bottom of the duct. In dust-collection equipment back mixing against even larger separating forces limits collection efficiency. It is suggested that irregular particle bounce is a major mechanism for such back mixing.

Figure 5 shows the concentration gradient for sand particles of angular shape conveyed horizontally in a rectangular duct. The sand particles were about 90 μ in average diameter (range 74 to 105 μ) and had a terminal falling velocity of about 1.5 ft./sec. The duct was 2 in. high by 5 in. wide by 16 ft. long, and the concentration gradient was measured at 14 ft. from the inlet, where there was nearly a uniform concentration. The central air velocity was 50 ft./sec. If all the particles were traveling horizontally when they entered the duct, one would expect to find them on the bottom of the duct after 5 ft. of duct length. By

Table 1. Concentration of Bouncing Particles

Average diameter, μ	Angular tin 50		Rounded tin 50		Angular sand 93	
Sp. gr.	7.3		7.3		2.64	
$1/\delta$, deg.	3.5		1.8		3.6	
y, ft.	Observed concentration, g./sq. ft.	Calculated concentration, g./cu. ft.	Observed concentration, g./cu. ft.	Calculated concentration, g./cu. ft.	Observed concentration, g./cu. ft.	Calculated concentration, g./cu. ft.
0.0075	7.0	5.35	7.64	3.38	2.54	2.63
0.0256	4.37	4.38	2.62	2.07	1.75	1.75
0.054	3.35	3.33	1.00	0.994	1.099	1.08
0.0767	2.5	2.56	0.437	0.43	0.654	0.627
0.102	1.95	1.98	0.18	0.207	0.398	0.397
0.1275	1.42	1.515	0.093	0.0925	0.259	0.219
0.1458	—	1.255	0.067	0.051	0.204	0.167

the time the particles had traveled 14 ft. along the duct, where the concentrations of Figure 5 were measured, they must have undergone multiple bounces and were approaching some equilibrium bounce state. In accordance with the Weiss-Longwell criterion (2), turbulent diffusion could not have been the cause of back mixing.

SIMPLE THEORY FOR EQUILIBRIUM CONCENTRATION OF BOUNCING PARTICLES

A particle striking a surface at an incident angle θ with a certain velocity and initial rotation will bounce at some reflected angle $(\theta + \alpha)$ with a new velocity and rotation which is a function of the elasticity of bounce, the irregularity of the particle, and the friction encountered on impact. α consists of two parts; the first part establishes the average angle of reflection, and the second part involves a probability of dispersion about the average angle of reflection. After rebound a certain portion of the particles will have an increased lateral velocity which will carry them a large distance from the rebound surface.

To obtain the order of back mixing by bounce, a simplified system was considered. In this simplified system it is assumed that the incident angle is quite large ($\theta = 90$ deg.), that the particle bounces at the angle α to the surface, and that the particle hits at the main-stream velocity and bounces at the same velocity (no energy loss or interchange of energy). It is also assumed that the angle α is distributed normally about an average reflection angle of 90 deg., dispersion through the surface being perfectly reflected away from the surface, such that

$$(2\delta/\sqrt{\pi})\exp(-\delta^2\alpha^2)\,d\alpha$$

represents the fraction of the total number of bouncing particles which bounce at a positive angle between α and $d\alpha$.

The velocity away from the surface becomes

$$v_y = v_o \sin \alpha \approx v_o\alpha \qquad (9)$$

where α is nearly always less than 15 deg. It is assumed further that the stopping distance of a particle bouncing at an angle α is governed by Stokes's resistance law.

At every y distance above the surface, particles falling toward the surface are assumed to fall with their terminal velocity. This is certainly not true of particles whose bounce carries them only to this level, but the concentration at this level is made up primarily of particles raining down from much higher levels.

At equilibrium a flux balance at a certain level reads

$$CV_t = \int_{\alpha}^{\infty} C_w V_t (2\delta/\sqrt{\pi})\exp(-\delta^2\alpha^2)\,d\alpha \qquad (10)$$

where the first term represents the flux toward the wall caused by the separating force, and the integral term represents the flux away from the surface of those particles which have stopping distances greater than the y level in question. C_wV_t is the flux of particles into the surface; $(2\delta/\sqrt{\pi})\exp(-\delta^2\alpha^2)\,d\alpha$ is the fraction of particles bouncing at an angle α, reaching a stopping distance $S_s = y$. Rearrangement of the flux balance gives

$$C/C_w = 1 - \text{erf}(q) = 1 - (2/\sqrt{\pi})\,[q-(q^3/3.1!)+(q^5(5.2!)-\ldots] \qquad (11)$$

where $q = \alpha\,\delta$.

The effective diffusion coefficient becomes

$$\mathcal{D} = V_tC/(dC/dy) \qquad (12)$$

Since $\mathcal{D}$ cannot be obtained explicitly in terms of y, the effective diffusion coefficient must be obtained by graphical differentiation of a plot of $C = C(y)$.

It is noted finally in this theoretical development that back mixing can also occur because the particles are bouncing from randomly oriented units of bounce surface area. In this case the irregularity of the surface is of a scale which may seem small to the eye but is large with respect to the size of the particles. The mathematical treatment and theoretical results will be the same, but now α is a function of, and in special cases may be only a function of, surface characteristics.

ANGULAR DEVIATIONS OF REFLECTED ANGLES CALCULATED FROM CONCENTRATION GRADIENTS

If the simple theory for equilibrium concentration of bouncing particles is substantially correct, the angular deviation of bounce can be calculated from concentration gradients such as the one shown in Figure 5. For a point of known y distance from the wall the concentration was obtained and placed in Equa-

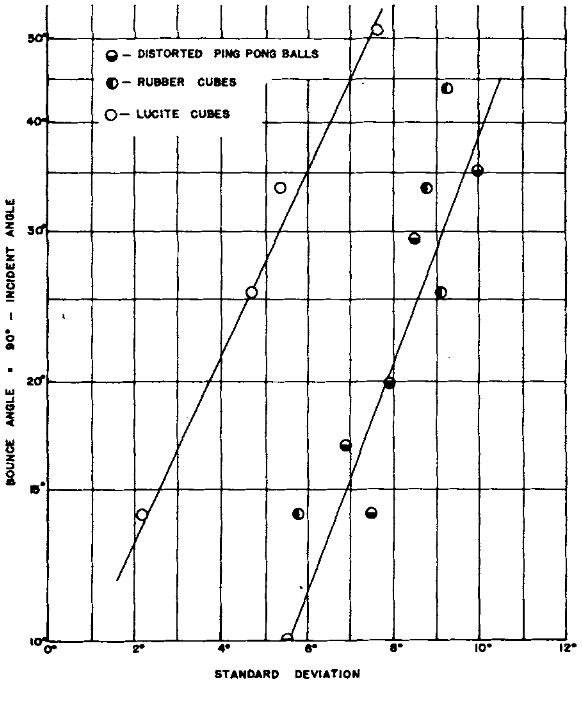

Fig. 3. Dispersion about the average bounce angle is affected by incident angle and type of contact surfaces.

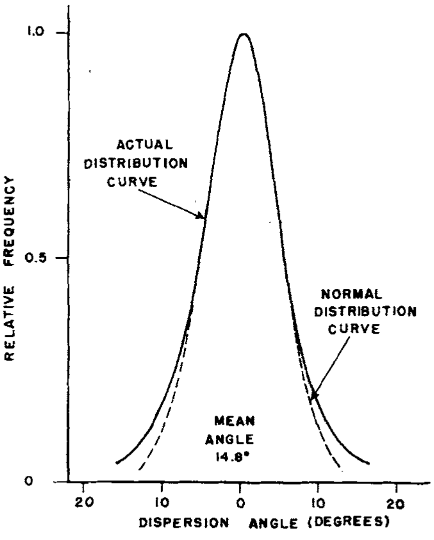

Fig. 4. Dispersion about the average angle of reflection is normal in character.

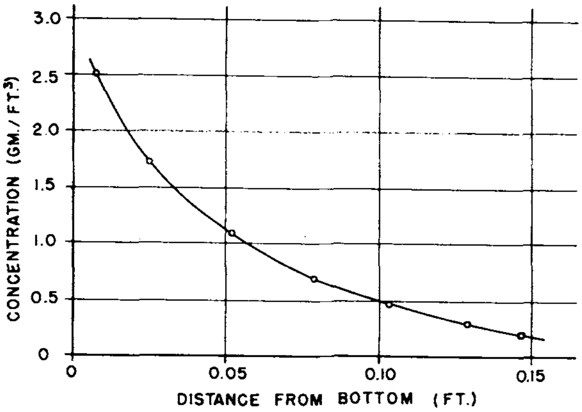

Fig. 5. Example of sand-concentration distribution maintained by bouncing particles.

tion (11). The known y represented a stopping distance related to the angle α which was required to bounce a particle to that height. A similar procedure was followed for a second point. Two equations were now available with the unknowns C_w and q, where $q_1 = \alpha_1\delta$ and $q_2 = \alpha_2\delta$. By dividing the resulting equations, an expression for δ may be obtained for each type of particulate matter in question. Once δ is established, C_w and the concentration at other levels can be calculated. It is to be noted that δ is constant for a given material, but α varies from level to level and is related to the stopping distance.

Results of such calculations for three different types of particles are shown in Table 1. It is noted that the standard-deviation angle for angular sand and tin is nearly that which would be predicted from an extrapolation of Figure 3 to the lower incident angles of these cases. As expected, the deviation angle for rounded tin particles is much less than that for angular particles.

SIMPLE THEORY FOR THE EFFECT OF IRREGULAR BOUNCE ON ENERGY LOSSES DURING PNEUMATIC CONVEYING

Using the assumptions and symbols of the section on back mixing, one can also consider the effect of irregular bounce on energy losses during the horizontal conveying of particle suspensions. It remains only to assume that kinetic energy represented by lateral velocities is lost energy and is dissipated in particle wakes in the gas volume above the bounce surface.

The energy loss rate per unit area of bounce surface is

$$\int_0^\infty (C_w V_t (v_o\alpha)^2/2g_c)\ (2\delta/\sqrt{\pi})\exp(-\delta^2\alpha^2)d\alpha = C_w V_t v_o^2/2g_c\delta^2 \quad (13)$$

If this energy is supplied by the gas moving over the surface, the gas will experience, in addition to other energy losses, an energy loss caused by the irregular bounce. In tube flow, when one assumes a floor area of DL, this energy loss per unit mass of gas flowing is given by

$$N_f = (F/L)D/(v_o^2/2g_c) = (4/\pi)(C_w/\rho)(v_o)(1/\delta^2) \quad (14)$$

For $C_w/\rho = 1$ lb.-mass solids/lb.-mass gas, $V_t/v_o = 10^{-1}$, the ratio of terminal falling velocity to flow velocity, and $1/\delta = \pi/20$ radians; $N_f = 3 \times 10^{-3}$, which is of the order of 10 to 20% of that expected for turbulent gas flow alone.

The preceding calculation is significant only so far as it suggests a method of attack on a rather complex problem. The role and significance of friction at the moment of impact and interchange of energy between translation and rotation have not been considered. C_w, the effective particle concentration in the gas near the wall, is usually unknown. The interference between particles has been neglected.

In the usual case of particle transport by bounce the average particle velocity is somewhat less than average gas velocity (3), an indication that bouncing particles are always lagging behind the gas flow. The combined particle wakes for the relative axial velocity difference also produce an energy loss in the gas. Since the measured velocity lag would result in a much more significant energy loss than that for lateral motion alone, it is presumed that the importance of irregular bounce lies primarily in its effect on the lag, or slip, velocity. However such a theoretical derivation and experimental verification is beyond the simple theory and experiment of the present paper.

ACKNOWLEDGMENT

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NOTATION

C = suspended particle concentration in gas flow
C_w = suspended particle concentration in gas flow at bounce surface
$\mathbf{C}$ = vector velocity of center of mass of stick
E = total energy of stick
F = energy loss per unit mass of gas flowing
g_c = dimensional conversion constant = 32.2 (lb.mass/lb.-force)(ft./sec.^{2})
$\underline{\mathbf{I}}$ = inertia tensor of bouncing stick
$\mathbf{J}$ = impulsive change in momentum
$2\mathcal{L}$ = length of bouncing stick
m = mass of bouncing stick
q = $\alpha\,\delta$
S_s = stopping distance of particle if Stokes resistance law pertains
v_o = gas-flow velocity, main stream
V_t = terminal falling velocity of particle
$\mathbf{x}, \mathbf{y}$ = unit vectors in x and y directions
y = vertical distance from bounce surface

Greek Letters

α = angular deviation of the bounce, measured from the average angle of reflection
$1/\delta$ = standard deviation of the angular deviation of the angle of reflection
θ = angle of incidence measured from normal to bounce surface
θ' = angle of reflection
ρ = gas density
ϕ = angle between stick and surface at moment of impact
$\underline{\omega}$ = rotation of bouncing stick

Subscripts x, y, z refer to vector components in x, y, z directions; primed quantities denote conditions after collision, unprimed quantities conditions before collision.

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